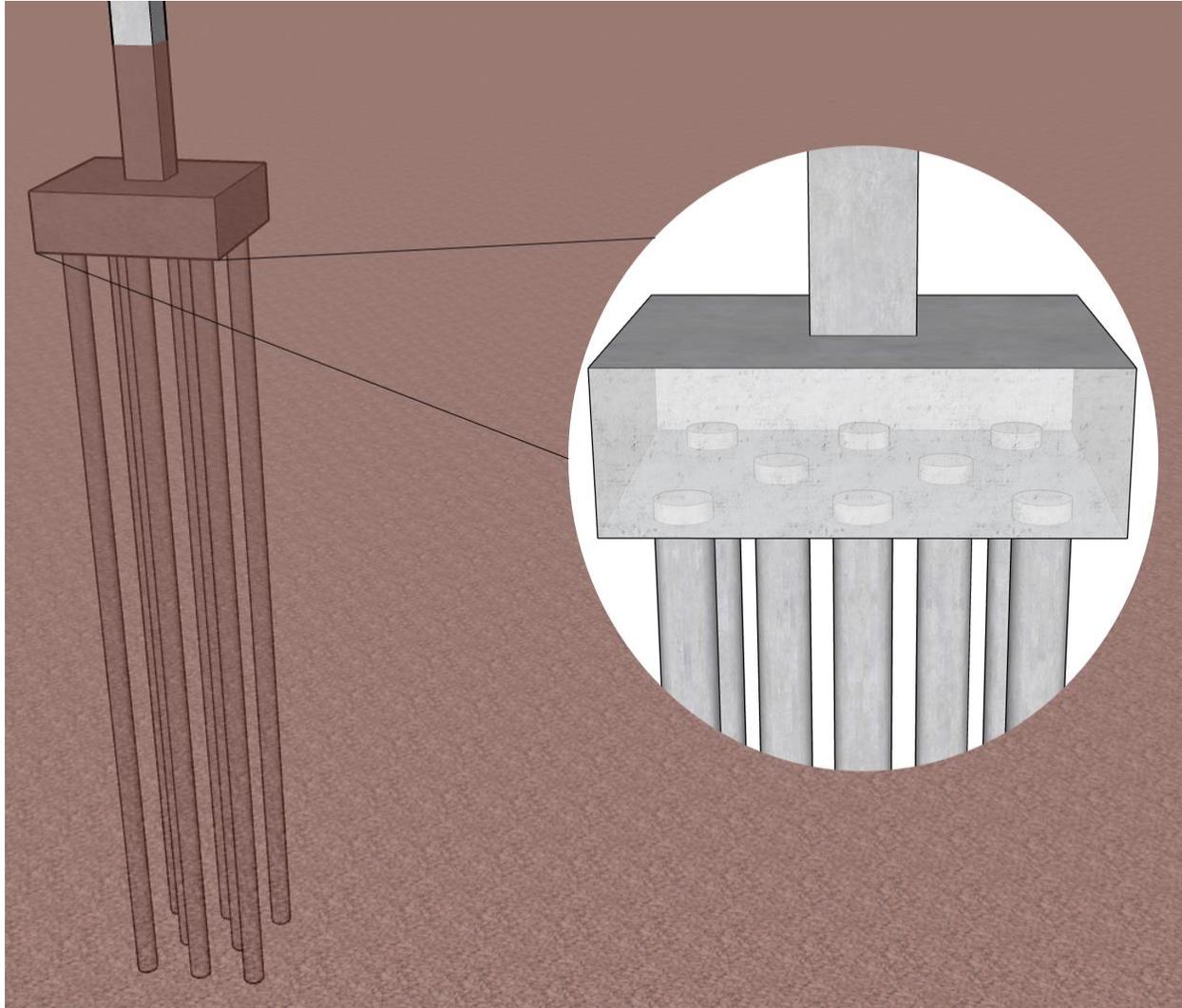


Pile Supported Foundation (Pile Cap) Analysis and Design



Pile Supported Foundation (Pile Cap) Analysis and Design

Based on a geotechnical study, a pile supported foundation is required to support a heavily loaded building column. Design the pile cap shown in the following figure with 12 in. diameter piles and a service load capacity of 50 tons each. The pile cap has normal-weight concrete with a compressive strength of 4000 psi and Grade 60 reinforcement. And the piles are embedded 4 in. into the pile cap. The axial loads on the column are due to dead and live loads and are equal to 425 and 250 kips, respectively. The following hand solution will be used for a comparison with the Finite Element Analysis (FEA) and design results of the engineering software program [spMats](#).

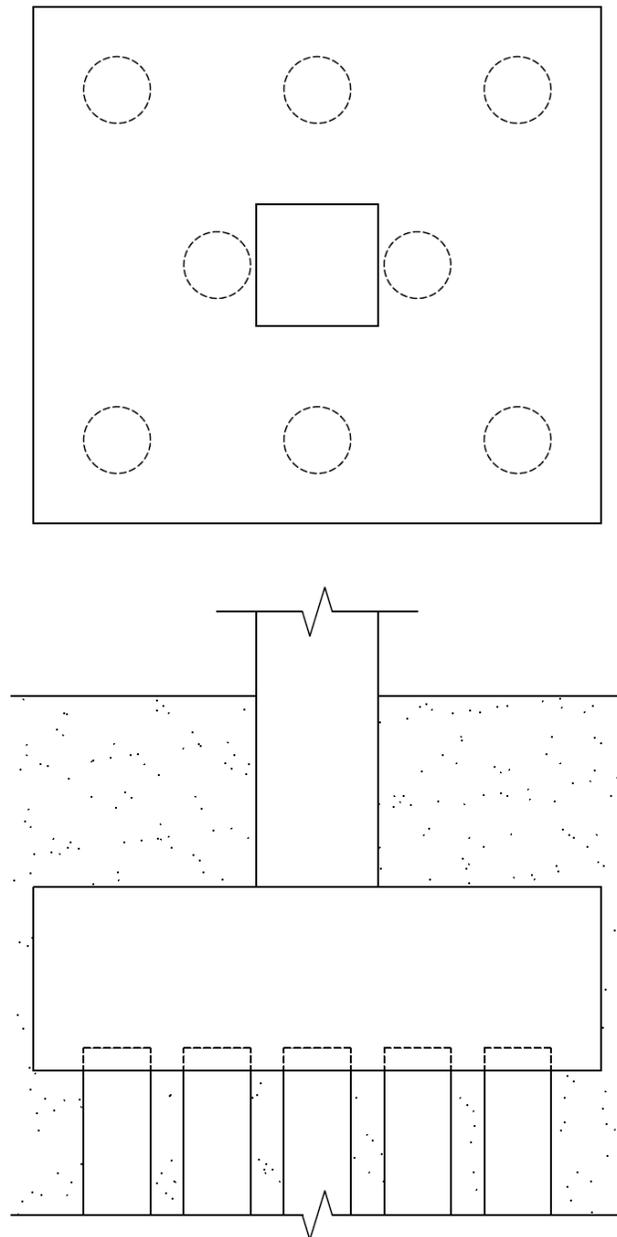


Figure 1 – Reinforced Concrete Pile Cap

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Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Reinforced Concrete Structures, 2nd Edition, 2016, David A. Fanella, McGraw-Hill Education, Example 10.13
spMats Engineering Software Program Manual v8.50, StructurePoint LLC., 2016

Design Data

$f_c' = 4,000$ psi normal weight concrete

$f_y = 60,000$ psi

Dead load, $D = 425$ kips

Live load, $L = 250$ kips

Pile Diameter, $d_p = 12$ in.

Piles are embedded 4 in. into the pile cap

1. Pile Cap Plan Dimensions

The plan dimensions of a pile cap depend on the number of piles that are needed to support the load. Number and arrangement of piles were determined from unfactored forces and moments transmitted to piles and permissible pile capacity selected through geotechnical engineering principles. These calculations are not part of the scope of this example. *ACI 318-14 (13.4.1.1)*

Pile spacing is generally a function of pile type and capacity. The following figure shows the piles configuration selected by the reference for this example with the associated pile cap plan dimensions.

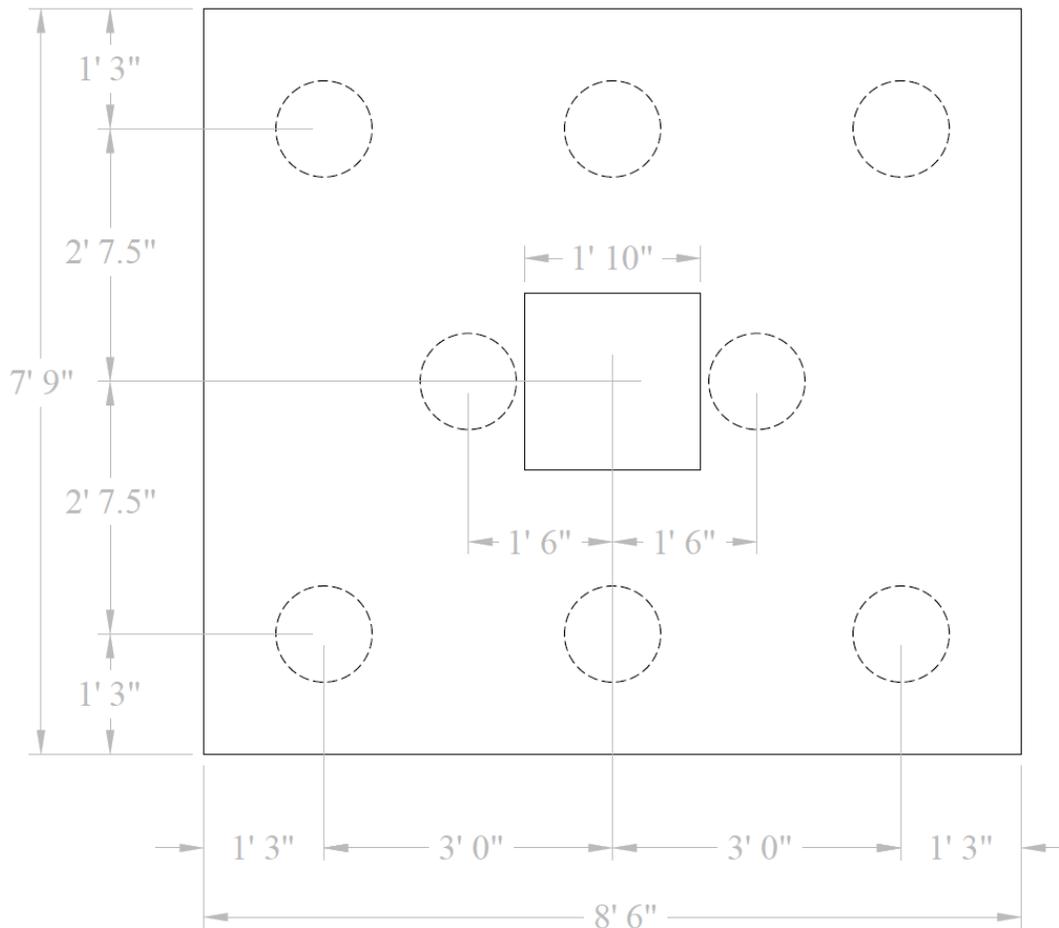


Figure 2 – Pile Cap Plan Dimensions

With the approximate plan dimensions determined based on a suitable number of piles for the applied design loads, the determination of foundation thickness (depth) can be completed by examining shear and flexural strength at critical sections. The pile configuration selected in the reference is intended to illustrate the implementation of code provisions in 13.4.2.5 pertaining to the calculation of factored shear based on pile location with respect to critical shear section.

2. Pile Cap Shear Strength and Thickness

The thickness of the pile cap is initially determined based on the following key parameters:

- 1) Two-way shear requirements around the column,
- 2) One-way shear requirements at the column, and
- 3) One-way and two-way shear requirements around the piles
- 4) Pile cap overall depth minimum requirements

Both one-way and two-way shear are investigated at the critical sections around the vertical elements (columns) supported by the pile cap with the requirements of ACI 318-14 (13.4.2). Factored loads are used to compute the shear forces at the critical sections around columns as well as piles. Like footings, it is common practice not to use shear reinforcement in pile caps. One-way shear checks are shown for illustration as two-way shear normally governs the design. The overall depth of pile cap shall also be selected such that the effective depth of bottom reinforcement is at least 12 inches.

2.1. Two-Way Shear at the Column

The total factored axial load on the column:

$$P_u = 1.2 \times P_D + 1.6 \times P_L = 1.2 \times 425 + 1.6 \times 250 = 910 \text{ kips}$$

Consider a uniform distribution of column load to the provided piles. Because of the rigidity of the pile cap, this assumption will be verified later using finite element analysis in [spMats](#).

$$P_u \text{ per pile} = \frac{910}{8} = 114 \text{ kips (Assume this is acceptable given pile capacity given in the reference)}$$

Assuming that the centers of the two piles closest to the column are located $d_p/2 = 6$ in. or more inside the perimeter of the critical section for two-way shear, their contribution to the total shear can then be neglected. This assumption will be verified later. **ACI 318-14 (13.4.2.5(b))**

Therefore, the factored shear force at the critical section is calculated based on six of the eight piles only:

$$V_u = 6 \times 114 = 684 \text{ kips}$$

The design shear strength for the interior column:

$$\phi V_c = \phi \times 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d \quad \text{ACI 318-14 (22.6.5.2(a))}$$

$$\phi V_c = 0.75 \times 4 \times 1.0 \times \sqrt{4000} \times (4 \times (22 + d)) \times d = 16,697 \times d + 759 \times d^2$$

Where $\phi = 0.75$ **ACI 318-14 (Table 21.2.1)**

The minimum required d for the pile cap foundation can be calculated by setting $V_u = \phi V_c$ as follows:

$$\phi V_c = V_u$$

$$759 \times d^2 + 16,697 \times d - 684,000 = 0$$

$$d = 21 \text{ in.}$$

By using $d = 21$ in., the center of the two piles closest to the column is located $(43/2) - 18 = 3.5$ in. inside the critical section which less than $d_p/2 = 6$ in. (see the following figure). Thus, the initial assumption is not valid. That is part of the reactions from these two piles must be considered in the shear force at the critical section, or a bigger d must be selected.

By increasing d to 26 in., the center of the piles is located at $d_p/2$ inside the critical section, and these reactions can be neglected and the assumption made earlier is validated (see the following figure). These configuration changes are for illustration only and may not represent optimal design which is left up to the design professional based on project criteria.

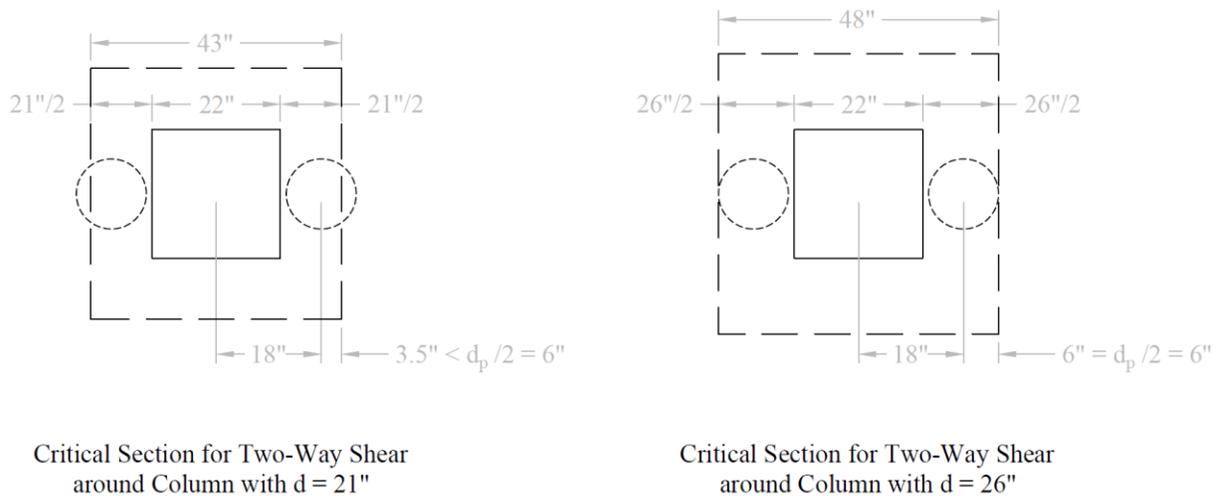


Figure 3 – Critical Sections for Two-Way Shear with Different d Values

The design shear strength for the interior column:

$$\phi V_c = \phi \times 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d$$

ACI 318-14 (22.6.5.2(a))

$$\phi V_c = 0.75 \times 4 \times 1.0 \times \sqrt{4000} \times (4 \times (22 + 26)) \times 26 = 947 \text{ kips} > V_u = 684 \text{ kips} \rightarrow o.k.$$

Thus, Two-way shear requirement around the column is satisfied.

2.2. One-Way Shear at the Column

North/South Direction:

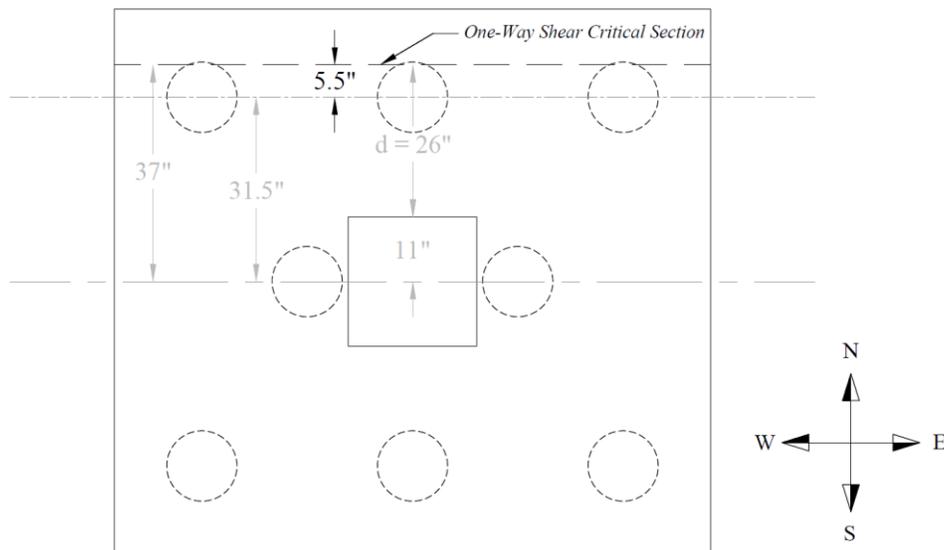


Figure 4 – One-Way Shear Critical Section in the North/South Direction

The critical section for one-way shear is located at a distance d from the face of the column as shown in the previous figure. The center of the three piles is located 5.5 in. inside of the critical section, which is less than $d_p/2$ (see the previous figure).

In this case, the pile reaction shall be based on a linear interpolation between full value at $d_p/2$ outside the section and zero value at $d_p/2$ inside the section. **ACI 312-14 (13.4.2.5(c))**

$$d - \frac{d_p}{2} = 26 - \frac{12}{2} = 20 \text{ in.} \rightarrow \text{the reaction from each pile} = 0 \text{ kips}$$

$$d + \frac{d_p}{2} = 26 + \frac{12}{2} = 32 \text{ in.} \rightarrow \text{the reaction from each pile} = 114 \text{ kips}$$

By performing interpolation with the center of the three piles is located at $26 - 5.5 = 20.5$ in. from the face of the column:

$$\text{the reaction from each pile} = 114 \times \left(\frac{20.5}{32 - 20} - 1.667 \right) = 5 \text{ kips}$$

Therefore, $V_u = 3 \times 5 = 15$ kips.

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f'_c} \times L \times d$$

ACI 318-14 (22.5.5.1)

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{4000} \times (8.5 \times 12) \times 26 / 1000 = 252 \text{ kips}$$

$V_u < \phi V_c \rightarrow \text{o.k.}$

East/West Direction:

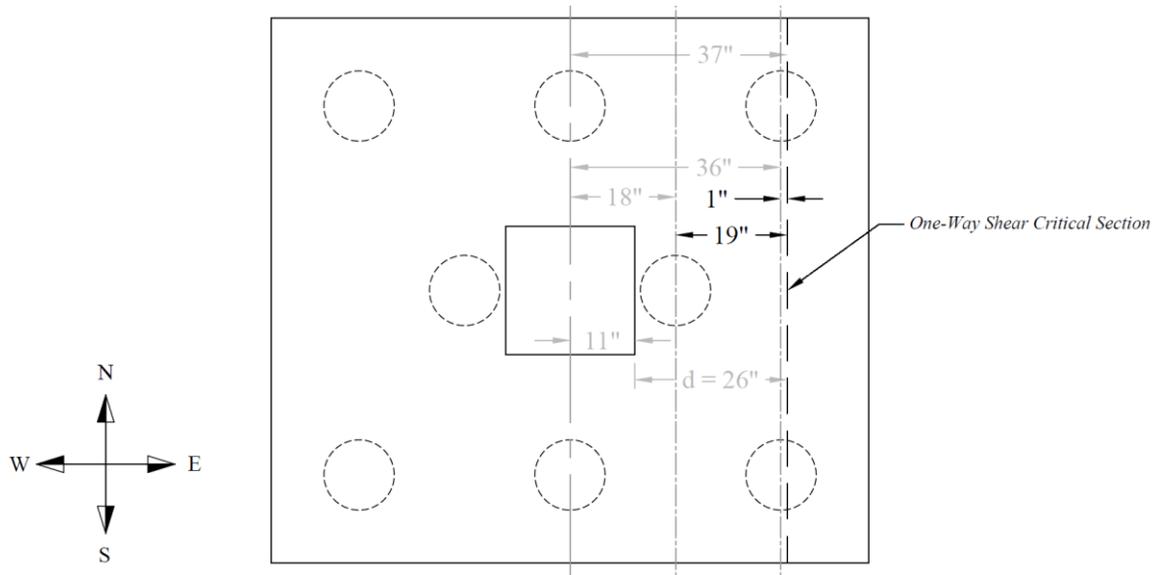


Figure 5 – One-Way Shear Critical Section in the North/South Direction

The critical section for one-way shear is located at a distance d from the face of the column as shown in the previous figure. The center of the pile closest to the column is located 19 in. inside of the critical section, which is greater than $d_p/2$ (see the previous figure). Thus the reaction from this pile is not considered.

The two remaining piles are located $26 - (36 - 11) = 1$ in. inside of the critical section, which is less than $d_p/2$. In this case, the reaction for those piles shall be based on a linear interpolation between full value at $d_p/2$ outside the section and zero value at $d_p/2$ inside the section. **ACI 312-14 (13.4.2.5(c))**

$$d - \frac{d_p}{2} = 26 - \frac{12}{2} = 20 \text{ in.} \rightarrow \text{the reaction from each pile} = 0 \text{ kips}$$

$$d + \frac{d_p}{2} = 26 + \frac{12}{2} = 32 \text{ in.} \rightarrow \text{the reaction from each pile} = 114 \text{ kips}$$

By performing interpolation with the center of the three piles is located at $26 - 1 = 25$ in. from the face of the column:

$$\text{the reaction from each pile} = 114 \times \left(\frac{25}{32 - 20} - 1.667 \right) = 48 \text{ kips}$$

Therefore, $V_u = 2 \times 48 = 96$ kips.

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f'_c} \times B \times d$$

ACI 318-14 (22.5.5.1)

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{4000} \times (7.75 \times 12) \times 26 / 1000 = 229 \text{ kips}$$

$V_u < \phi V_c \rightarrow \text{o.k.}$

2.3. Two-Way Shear at Corner Pile

$V_u = \text{Pile reaction} = 114 \text{ kips}$

$$\phi V_c = \phi \times 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d$$

ACI 318-14 (22.6.5.2(a))

Where $b_o = \pi (d_p + d)$ normally for an interior circular pile.

For the corner pile however, b_o is calculated as shown in the following figure:

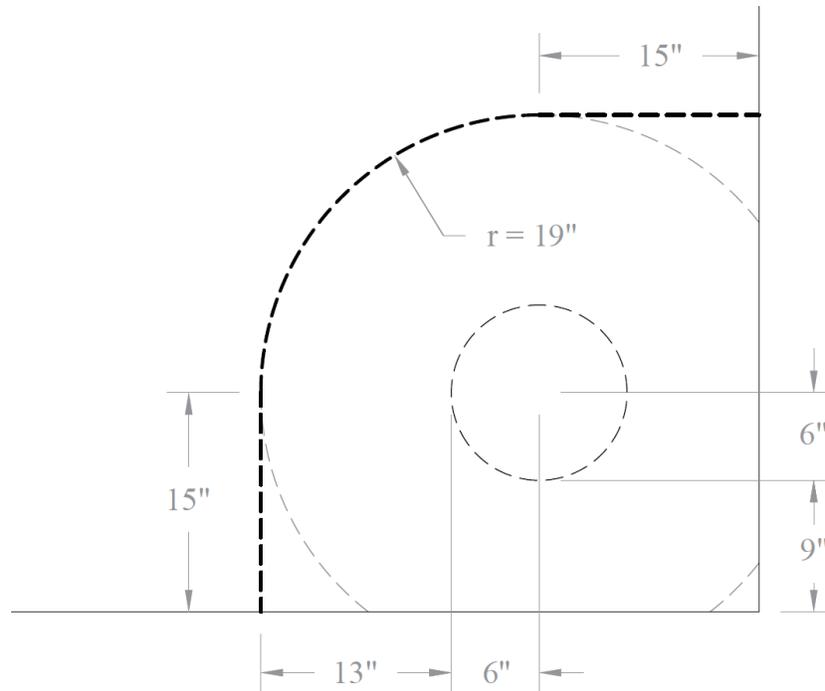


Figure 6 – Two-Way Shear Critical Section at Corner Pile

$$\phi V_c = 0.75 \times 4 \times 1.0 \times \sqrt{4000} \times \left[\frac{\pi}{4} \times (12 + 26) + (2 \times 15) \right] \times 26 / 1000 = 295 \text{ kips}$$

$V_u < \phi V_c \rightarrow \text{o.k.}$

The calculations above may be simplified using the approximation of equivalent sections as shown in ACI 8.10.1.3

2.4. One-Way Shear at Corner Pile

The one-way shear is checked at a distance d , but no more than 13 in., from the face of a corner pile as stated in the reference. The critical section occurs at a 45° angle with respect to the edges of the pile cap.

$V_u = \text{the reaction of the pile} = 114 \text{ kips}$

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f'_c} \times b \times d$$

ACI 318-14 (22.5.5.1)

Where b is calculated as shown in the following figure.

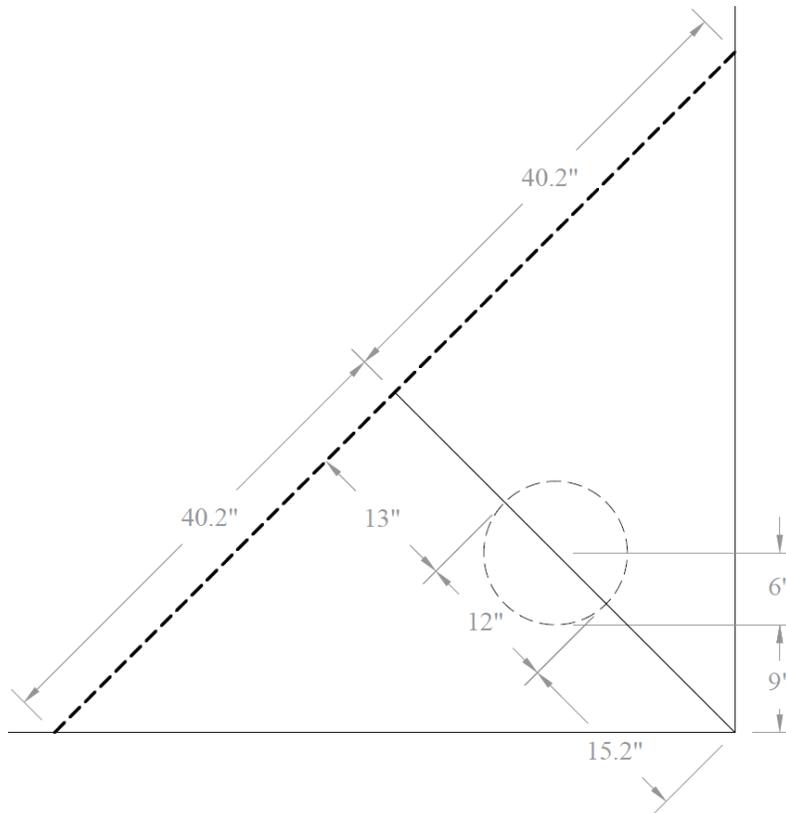


Figure 7 – One-Way Shear Critical Section at Corner Pile

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{4000} \times (2 \times 40.2) \times 26 / 1000 = 198 \text{ kips}$$

$$V_u < \phi V_c \rightarrow \text{o.k.}$$

2.5. Two-Way Shear at Interior Piles

For interior piles, if two-way shear perimeters overlap, the modified critical shear perimeter should be taken as that portion of the smallest envelope of individual shear perimeters that will actually resist the critical shear for group under consideration as shown in the following figure. ACI 318-14 (R13.2.7.2)

$$V_u = \text{Pile reactions} = 2 \times 114 = 228 \text{ kips}$$

$$\phi V_c = \phi \times 4 \times \lambda \times \sqrt{f'_c} \times b_o \times d$$

ACI 318-14 (22.6.5.2(a))

Where b_o is calculated as shown in the following figure.

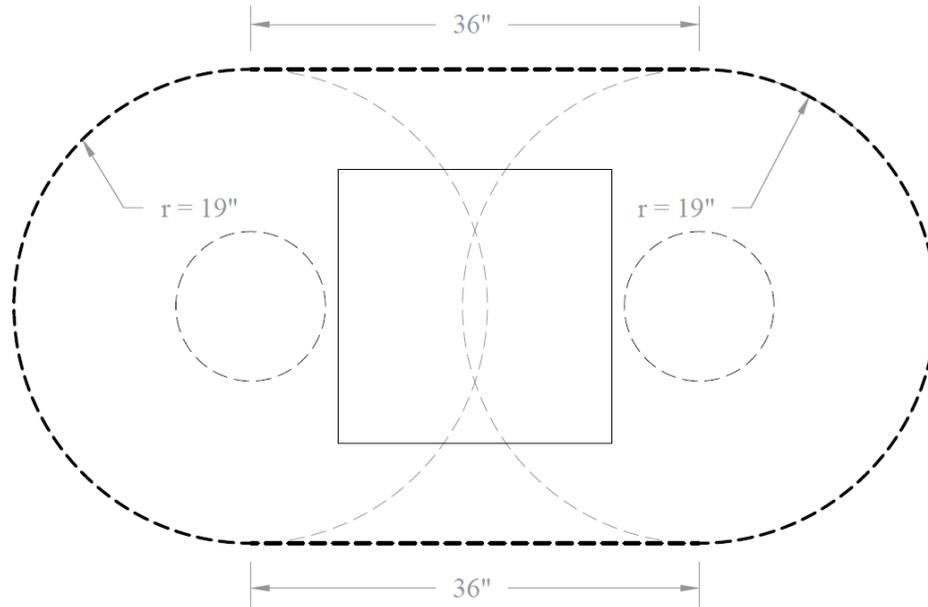


Figure 8 – Overlapped Two-Way Shear Critical Sections

$$\phi V_c = 0.75 \times 4 \times 1.0 \times \sqrt{4000} \times \left[\left(2 \times \left(\pi \times \left(\frac{12}{2} + \frac{26}{2} \right) \right) \right) + (2 \times 36) \right] \times 26 / 1000 = 944 \text{ kips}$$

$$V_u < \phi V_c \rightarrow \text{o.k.}$$

The reference states critical sections for two-way shear of the edge piles overlap and are incomplete (they fall outside of the pile cap) and shear requirements do not control for these piles.

2.6. Pile Cap Overall Depth Requirements

Overall depth of pile cap shall be selected such that the effective depth of bottom reinforcement is at least 12 in.

ACI 318-14 (13.4.2.1)

$$h_{\min} = 12 \text{ in.} + \text{cover} + \text{embedment} = 12 + 3 + 4 = 19 \text{ in.}$$

$$h = 26 + 3 + 4 = 33 \text{ in.} > h_{\min}$$

Thus, use a final pile cap thickness of 33 in.

3. **Pile Cap Flexural Strength and Reinforcement**

In a pile cap foundation design, flexural moments are evaluated in two orthogonal directions (M_x and M_y). To simplify design calculations, an average value of d is used to represent both reinforcement curtains in the x and y directions.

3.1. Factored Moments

Short Direction

The factored moment at the critical section is obtained by multiplying the reactions from three piles by the distance from the center of the piles to the critical section (at the face of the column) as follows:

$$M_{ux} = (3 \times 114) \times \left(\frac{7.75}{2} - 1.25 - \frac{22}{2 \times 12} \right) = 584.3 \text{ kips-ft}$$

Long Direction

The factored moment at the critical section is obtained by multiplying the reactions from three piles by the distance from the center of the piles to the critical section (at the face of the column) as follows:

$$M_{uy} = 114 \times \left(1.5 - \frac{22}{2 \times 12} \right) + (2 \times 114) \times \left(3 - \frac{22}{2 \times 12} \right) = 541.5 \text{ kips-ft}$$

3.2. Required Flexural Reinforcement

Short Direction

$$M_{ux} = 584.3 \text{ kips-ft}$$

$$d = 26 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the footing section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.983d$. The assumptions will be verified once the reinforcement area is finalized.

$$\text{Assume } jd = 0.983 \times d = 25.56 \text{ in.}$$

$$A_s = \frac{M_u}{\phi f_y jd} = \frac{584.3 \times 12000}{0.9 \times 60000 \times 25.56} = 5.08 \text{ in.}^2$$

$$\text{Recalculate 'a' for the actual } A_s = 5.08 \text{ in.}^2 \rightarrow a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5.08 \times 60000}{0.85 \times 4000 \times (8.5 \times 12)} = 0.879 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.879}{0.85} = 1.03 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c} \right) d_t - 0.003 = \left(\frac{0.003}{1.03} \right) \times 26 - 0.003 = 0.073 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{584.3 \times 12000}{0.9 \times 60000 \times (26 - 0.879/2)} = 5.08 \text{ in.}^2$$

$$A_{s,\min} = \text{Greater of } \left\{ \frac{0.0018 \times 60,000}{f}, 0.0014 \right\} \times b \times h \quad \underline{\text{ACI 318-14 (7.6.1.1)}}$$

$$A_{s,\min} = 0.0018 \times (8.5 \times 12) \times 33 = 6.06 \text{ in.}^2 > A_s$$

$$\therefore A_s = 6.06 \text{ in.}^2$$

$$s_{\max} = \text{lesser of } \left\{ \frac{3h}{18 \text{ in.}} \right\} = \text{lesser of } \left\{ \frac{3 \times 33 = 99 \text{ in.}}{18 \text{ in.}} \right\} = 18 \text{ in.} \quad \underline{\text{ACI 318-14 (7.7.2.3)}}$$

Providing 11#7 bars with $A_s = 6.60 \text{ in.}^2$ satisfies strength requirements.

For rectangular foundation, a portion of the total reinforcement ($\gamma_s A_s$) in the short direction shall be distributed uniformly over a band width equal to the length of short side of footing, centered on centerline of column. Remainder of reinforcement required in the short direction ($(1-\gamma_s)A_s$) shall be distributed uniformly outside the center band width of footing.

ACI 318-14 (13.3.3.3)

This requirement is also helpful to address the effects resulting from load concentration near the column and two-way behavior in foundation structures. This is a similar effect to what is required in elevated slab supported by columns, the FEA results at the end of this example will demonstrate this behavior and indicate where bars need to be concentrated according to reinforcement contours.

$$\gamma_s = \frac{2}{\beta + 1} = \frac{2}{\frac{8.5}{7.75} + 1} = 0.95 \quad \underline{\text{ACI 318-14 (Eq. 13.3.3.3)}}$$

Provide 10#7 bars at 9 in. on centers ($\leq s_{\max}$) in the center band, and add one additional bar outside of the center band. Thus, provide 12#7 in the short direction.

Long Direction

$$M_{ux} = 541.5 \text{ kips-ft}$$

$$d = 26 \text{ in.}$$

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the footing section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.983d$. The assumptions will be verified once the reinforcement area is finalized.

Assume $jd = 0.983 \times d = 25.56$ in.

$$A_s = \frac{M_u}{\phi f_y jd} = \frac{541.5 \times 12000}{0.9 \times 60000 \times 25.56} = 4.71 \text{ in.}^2$$

$$\text{Recalculate 'a' for the actual } A_s = 4.71 \text{ in.}^2 \rightarrow a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.71 \times 60000}{0.85 \times 4000 \times (7.75 \times 12)} = 0.894 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.894}{0.85} = 1.05 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c} \right) d_t - 0.003 = \left(\frac{0.003}{1.05} \right) \times 26 - 0.003 = 0.071 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{541.5 \times 12000}{0.9 \times 60000 \times (26 - 0.894/2)} = 4.71 \text{ in.}^2$$

$$A_{s,\min} = \text{Greater of } \left\{ \frac{0.0018 \times 60,000}{f} \right\} \times b \times h \quad \text{ACI 318-14 (7.6.1.1)}$$

$$A_{s,\min} = 0.0018 \times (7.75 \times 12) \times 33 = 5.52 \text{ in.}^2 > A_s$$

$$\therefore A_s = 5.52 \text{ in.}^2$$

$$s_{\max} = \text{lesser of } \left\{ \frac{3h}{18 \text{ in.}} \right\} = \text{lesser of } \left\{ \frac{3 \times 33 = 99 \text{ in.}}{18 \text{ in.}} \right\} = 18 \text{ in.} \quad \text{ACI 318-14 (7.7.2.3)}$$

Provide 10#7 in the long direction. These bars are spaced uniformly over the width of the pile cap.

4. Reinforcement Bar Development Length

Flexural reinforcement must be properly developed in a concrete foundation in order for the foundation to perform as intended in accordance with the strength design method. The concept of the development length is stated as follows: minimum lengths of reinforcement must be provided beyond the locations of peak stress (critical sections) in the reinforcement in order to fully develop the bars.

The development length for the pile cap is calculated as follows:

$$l_d = \left(\frac{3}{40} \times \frac{f_y}{\lambda \sqrt{f'_c}} \times \frac{\Psi_t \Psi_e \Psi_s}{c_b + k_{tr}} \right) \times d_b \quad \text{ACI 318-14 (Eq. 25.4.2.3a)}$$

$$l_d = \left(\frac{3}{40} \times \frac{60,000}{1.0 \times \sqrt{4,000}} \times \frac{1.0 \times 1.0 \times 1.0}{2.5} \right) \times \frac{0.875}{12} = 2.1 \text{ ft}$$

Where:

$\lambda = 1.0$ (Light weight modification factor: normal weight concrete) ACI 318-14 (Table 25.4.2.4)

$\Psi_i = 1.0$ (Casting position modification factor: less than 12 in. of fresh concrete placed below horizontal reinforcement) ACI 318-14 (Table 25.4.2.4)

$\Psi_e = 1.0$ (Epoxy modification factor: uncoated or zinc-coated reinforcement) ACI 318-14 (Table 25.4.2.4)

$\Psi_s = 1.0$ (Size modification factor: #7 and larger bars) ACI 318-14 (Table 25.4.2.4)

$\frac{c_b + k_{tr}}{d_b} = 2.5$ (Confinement term: shall not exceed 2.5) ACI 318-14 (Eq. 25.4.2.3b)

$$\frac{c_b + k_{tr}}{d_b} = \frac{3+0}{0.875} = 3.4 > 2.5 \rightarrow \frac{c_b + k_{tr}}{d_b} = 2.5$$

The provided bar length is equal to:

$$l_{d,provided} = \frac{7.75}{2} - \frac{22}{2 \times 12} - \frac{3}{12} = 2.7 \text{ ft} \geq l_d = 2.1 \text{ ft} \rightarrow \text{o.k.}$$

Because the available development length is greater in the long direction, the bars in that direction can be fully developed.

Note that transfer of forces between the column and the pile cap needs to be checked to ensure interface reinforcement is provided or the column vertical bars are adequately developed and lapped with foundation flexural reinforcement. ACI 318-14 (16.3)

5. Pile Cap Analysis and Design – spMats Software

[spMats](#) uses the Finite Element Method for the structural modeling, analysis and design of reinforced concrete slab systems or mat foundations subject to static loading conditions.

The slab, mat, or footing is idealized as a mesh of rectangular elements interconnected at the corner nodes. The same mesh applies to the underlying soil with the soil stiffness concentrated at the nodes. Slabs of irregular geometry can be idealized to conform to geometry with rectangular boundaries. Even though slab and soil properties can vary between elements, they are assumed uniform within each element. Piles are modeled as springs connected to the nodes of the finite element model. Unlike for springs, however, punching shear check is performed around piles.

For illustration and comparison purposes, the following figures provide a sample of the input modules and results obtained from an spMats model created for the reinforced concrete pile cap in this example.

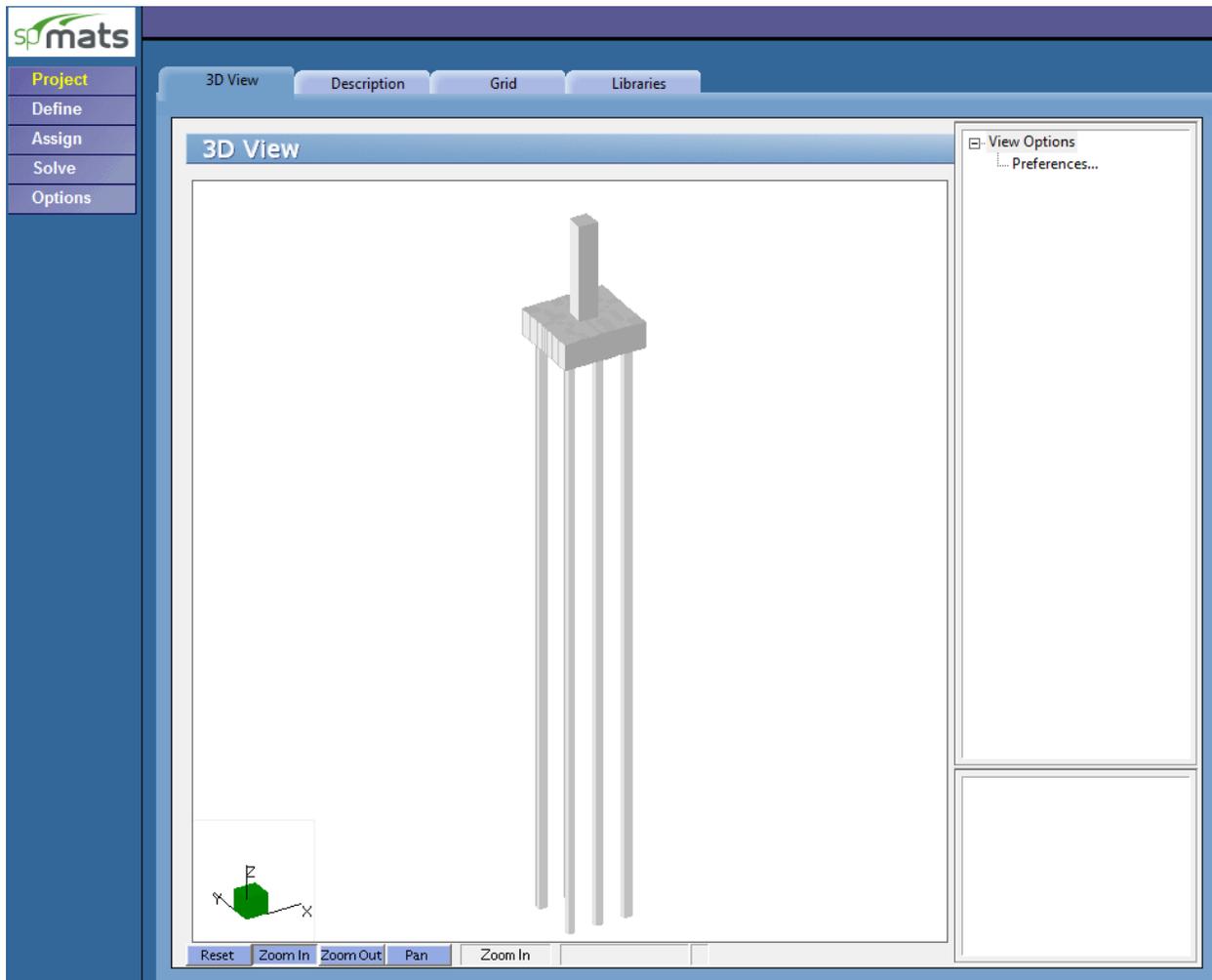


Figure 9 – 3D View for Pile Cap Foundation Model ([spMats](#))

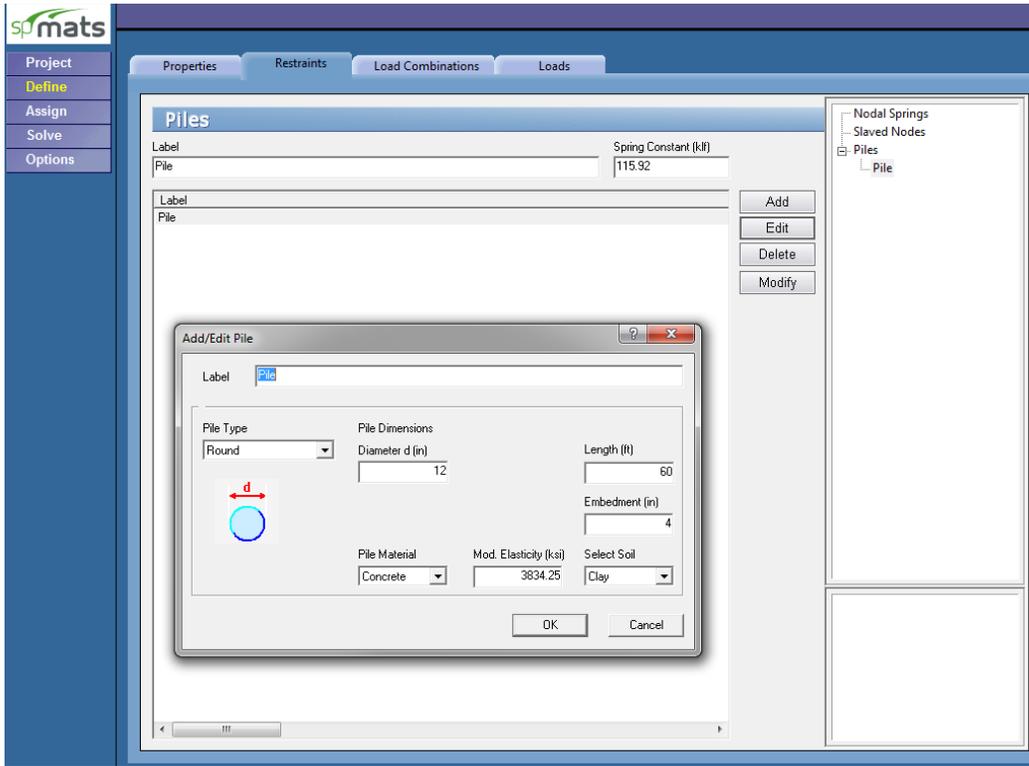


Figure 10 – Defining Piles (spMats)

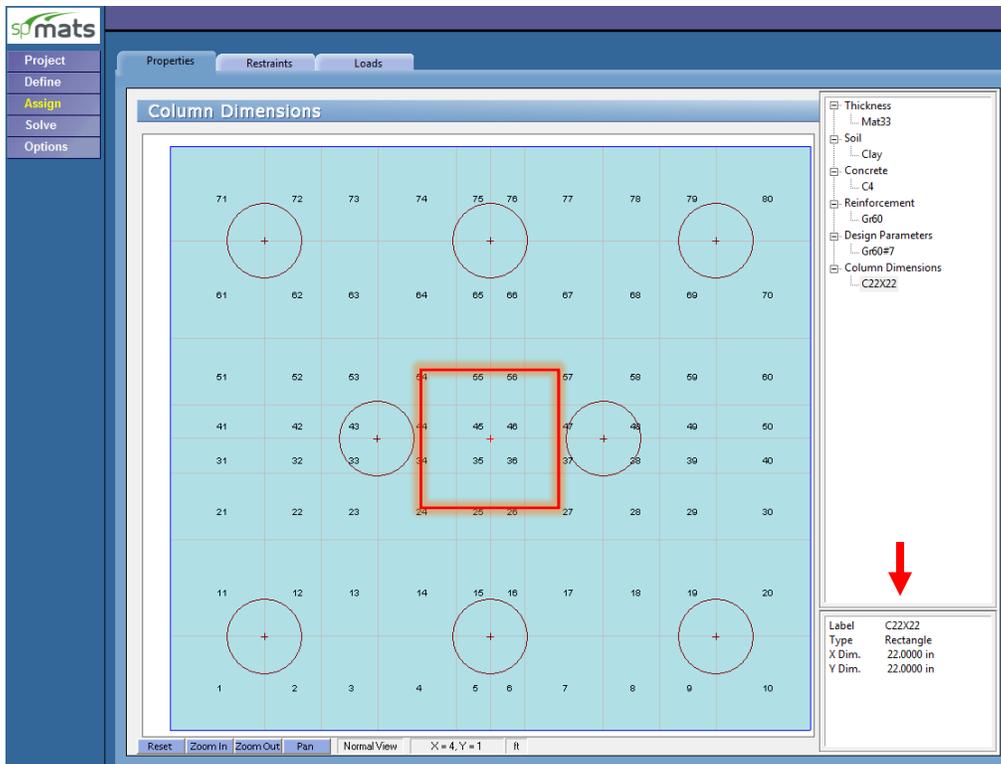


Figure 11 – Assigning Column (spMats)

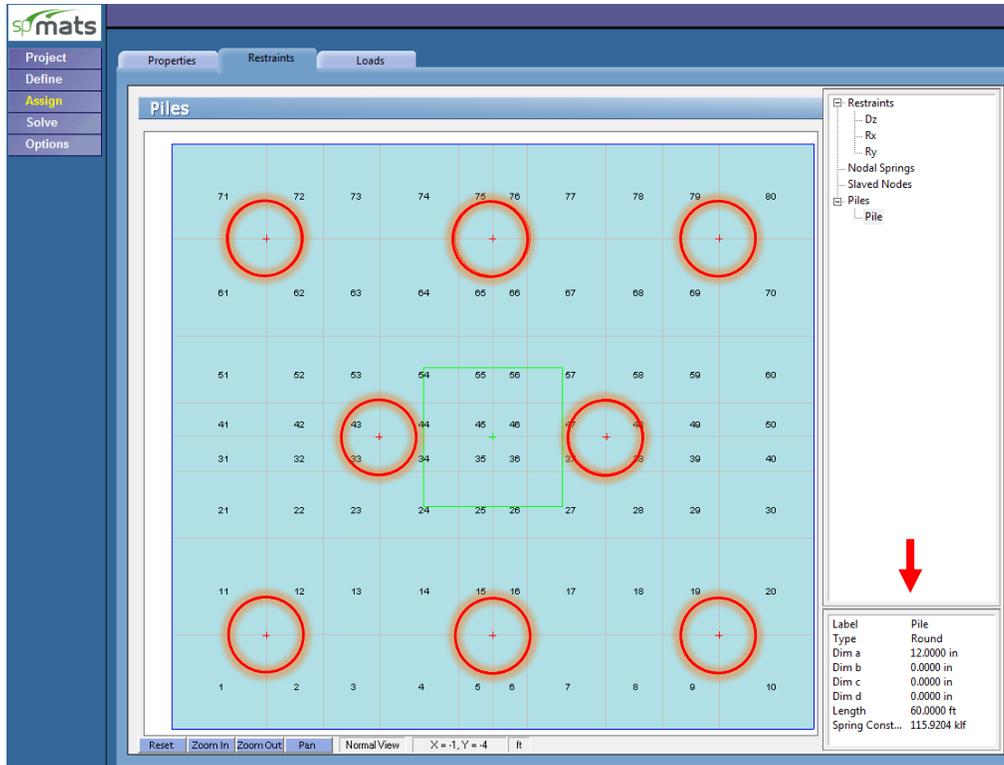


Figure 12 – Assigning Piles (spMats)

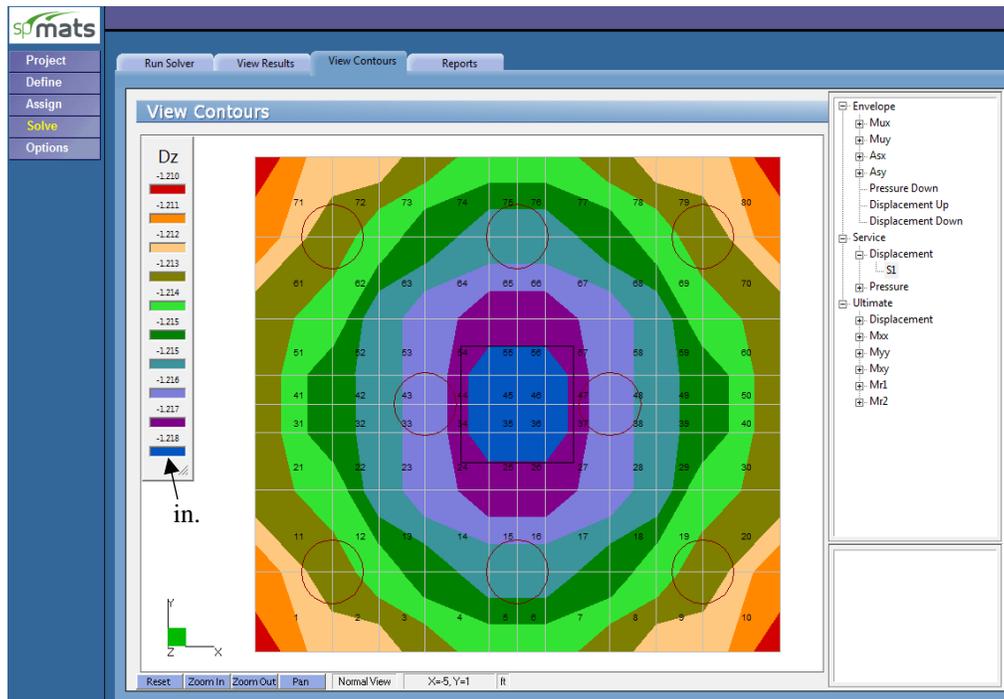


Figure 13 – Pile Cap Vertical Displacement Contour (spMats)

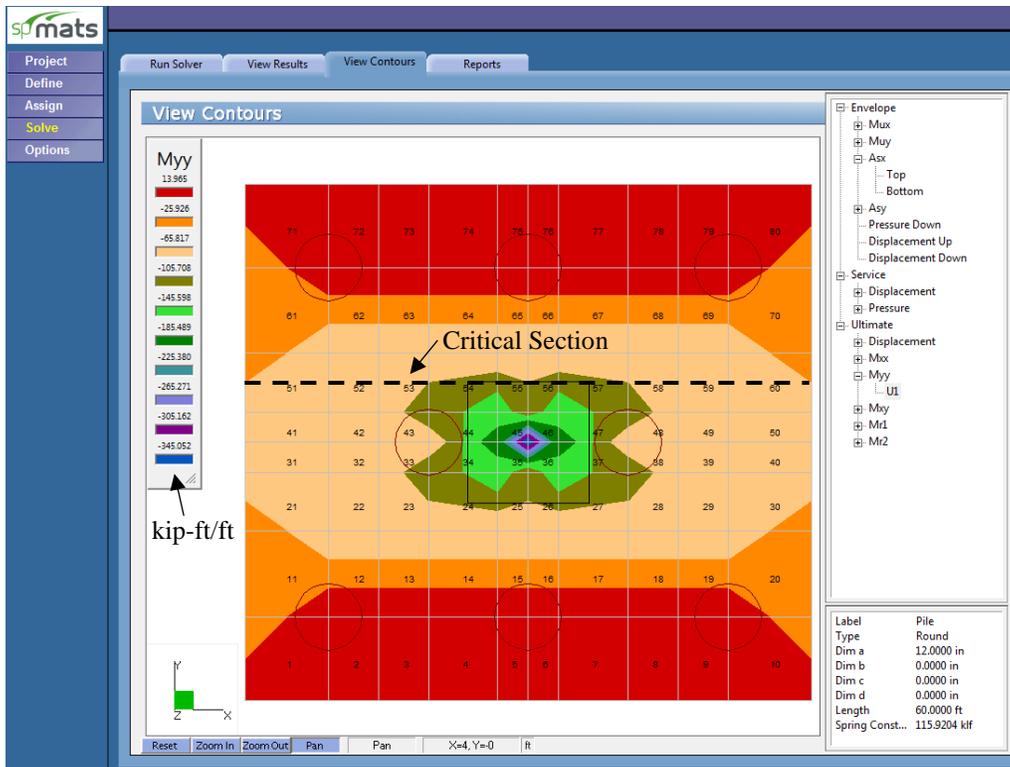


Figure 14 – Pile Cap Moment Contour along Y-Axis (spMats)

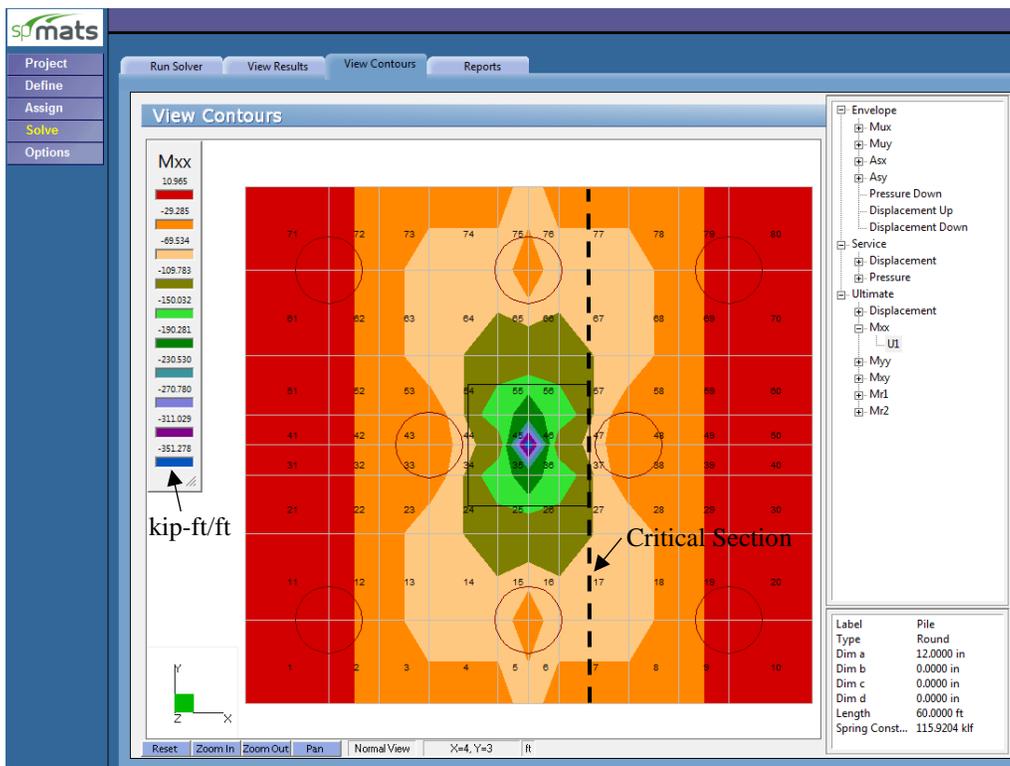


Figure 15 – Pile Cap Moment Contour along X-Axis - Complete Model (spMats)

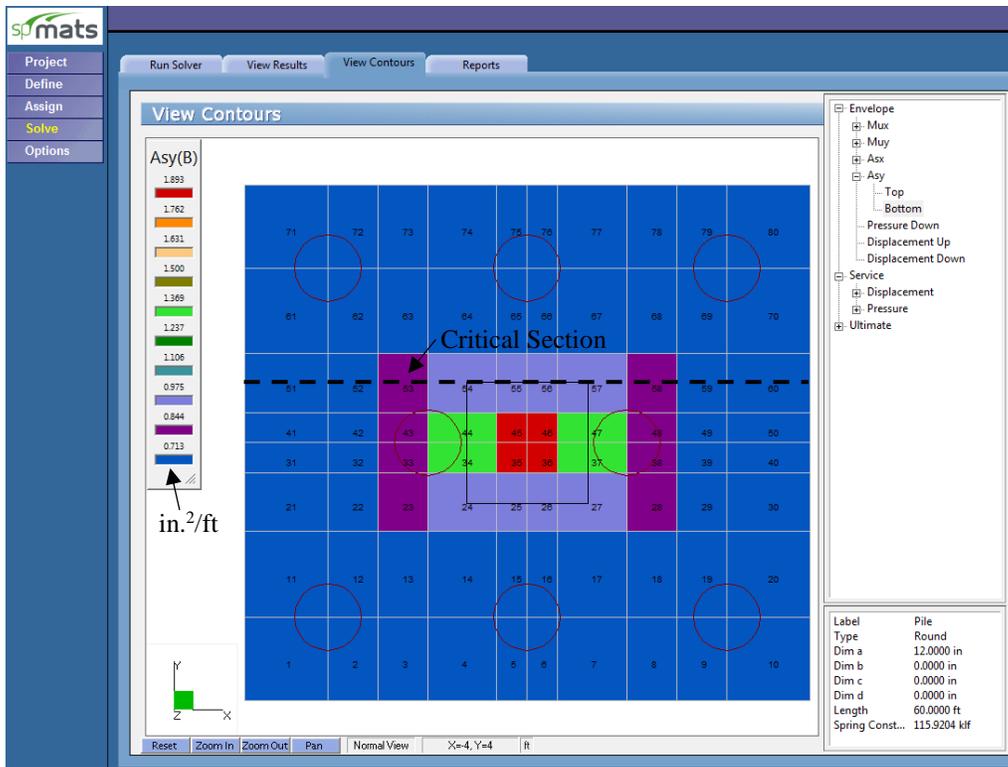


Figure 16 – Required Reinforcement Contour along Short Direction (spMats)

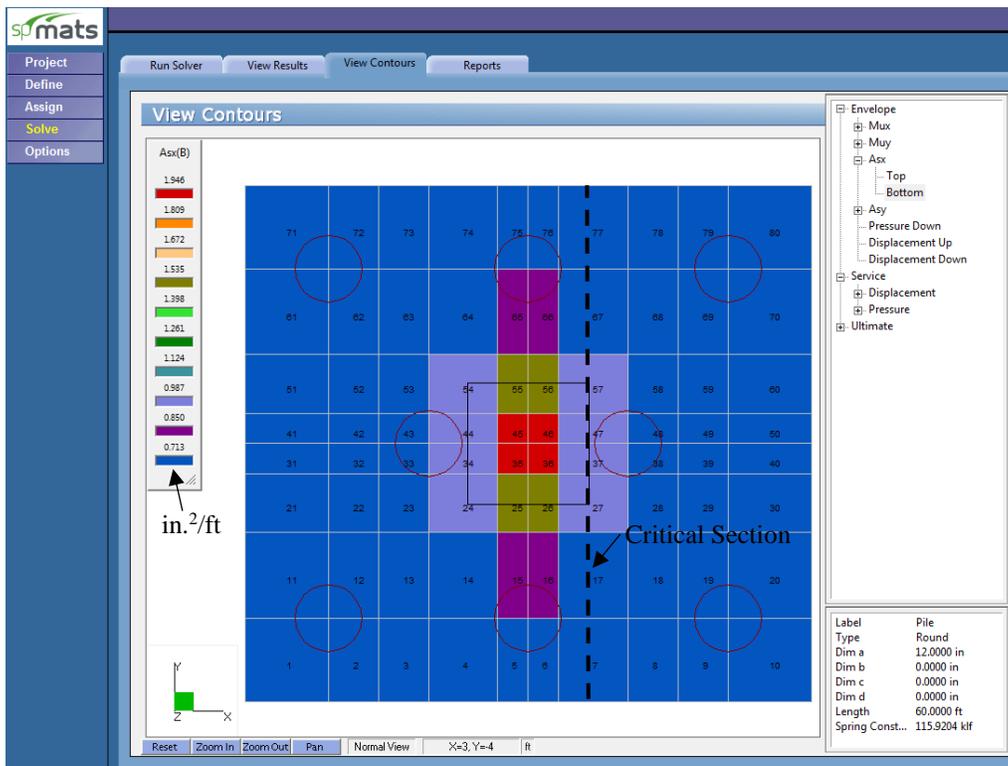


Figure 17 – Required Reinforcement Contour along Long Direction (spMats)

6. Design Results Comparison and Conclusions

6.1. Flexure

Solution	Short Direction		Long Direction	
	M_{design} (kips-ft)	$A_{s,required}$ (in. ²)	M_{design} (kips-ft)	$A_{s,required}$ (in. ²)
Hand	584.3	6.06	541.5	5.52
Reference	584.3	6.06	541.5	5.52
spMats (one-way action)	596.4	6.06 [†]	520.0	5.52 [†]
spMats (two-way action)	670.5	6.79	586.4	5.95

[†] For the design moment obtained from [spMats](#) considering one-way action, $A_{s,required}$ is calculated manually

The results of all hand calculations and the reference used illustrated above are in close agreement with the automated results obtained from the [spMats](#) FEA.

The difference in the design moment and required area of steel between the hand/reference and [spMats](#) is primarily a result of the analysis method used and can be related to differences in two key areas:

1. Design moment values:

In the hand/reference solution, the design is based on a single value for uniform design moment obtained along the critical section and the reinforcement required for this value is governed by the code minimum. In the FEA, the moment and the corresponding required reinforcement is distributed in accordance to the load demand and reflects the concentration around the column (figures 14-17). In other words, higher moments are presented in the elements closer to the column, leading to higher $A_{s,required}$ in those elements while the rest of the elements must still satisfy the $A_{s,min}$. [spMats](#) calculates the required area of steel for each element as a unit area per distance (in.²/ft) precisely addressing the moments where the demand is the highest where the majority of the load is concentrated. In this example the difference is 596 kips-ft (spMats) compared with 584 kips-ft (hand/reference).

2. Twisting moment contribution:

In practice, flexural reinforcement is generally provided in the orthogonal directions of the footing system and not in the principal directions. Therefore, the Principal of Minimum Resistance is used by [spMats](#) to obtain values for the design moments (M_{ux} or M_{uy}), which include the effects of the twisting moment (M_{xy}) in addition to the bending moment (M_{xx} or M_{yy}). In this example the difference is 670 kips-ft (spMats) compared with 584 kips-ft (hand/reference).

6.2. Shear

Table 2 – Comparison of Pile Cap Analysis and Design Results (Two-Way Shear)						
Solution	Around Column		Around Interior Piles		Around Corner Piles	
	v_u , psi	ϕv_c , psi	v_u , psi	ϕv_c , psi	v_u , psi	ϕv_c , psi
Hand	137.0	189.7	45.8	189.7	73.3	189.7
Reference	137.0	189.7	45.8	189.7	73.3	189.7
spMats	137.0	189.7	30.5	189.7	30.5	189.7

For two-way shear check around column:

Using provision ACI 318-14 (13.4.2.5(b)), the hand and reference solutions neglected the contribution of two piles near the column. Reflecting this condition in [spMats](#) can be easily implemented by excluding the same two piles from the model if required as shown in the following figure.

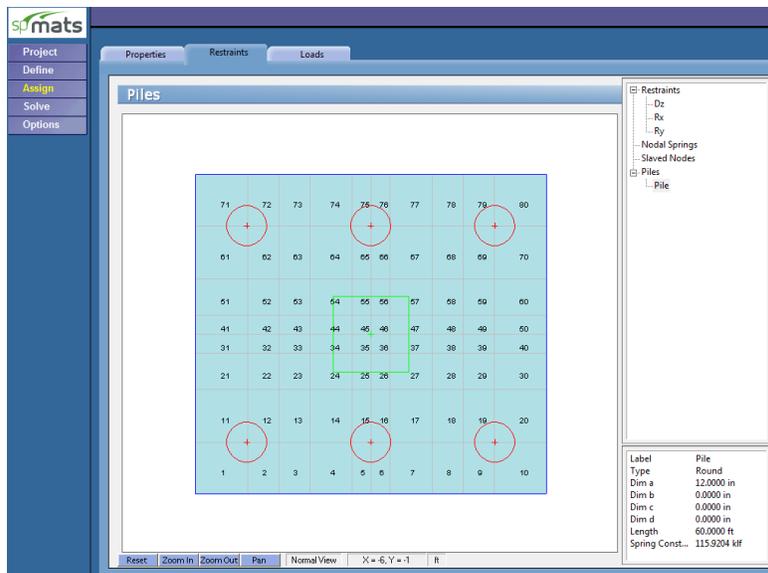
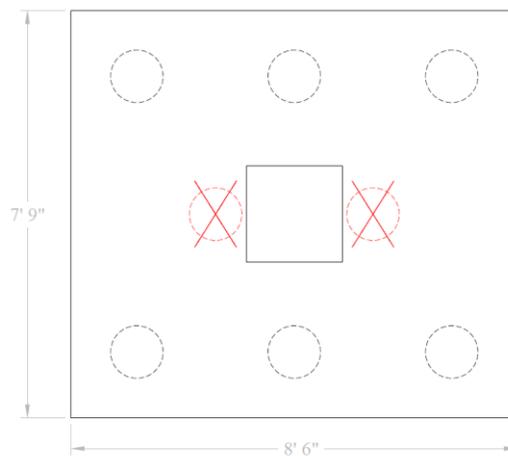


Figure 18 – Excluding the Two Piles Closer to the Column ([spMats](#))

Punching Shear							
B6 - Punching Shear Around Columns (Ultimate Load Combinations):							
Units --> Applied Shear Force Uu (kips), Applied Moments Mux, Muy (k-ft) Factored Shear Stress uu (psi), Factored Shear Resistance Phi*vc (psi) Concrete Strength f'c (psi), distances X_Offset, Y_Offset (ft) Average depth (in), Dimensions Bx, By (ft) Area (in^2), Jxx, Jyy, Jxy (in^4)							
Geometry of Resisting Area							
Node	Column Label	Location	Average Depth	Dimensions Bx	By	Centroid X_Offset	Y_Offset
50	C22X22	Inner	26.00	4.00	4.00	0.00	0.00
Properties of Resisting Area							
Node	Column Label	Area	Jxx	Jyy	Jxy		
50	C22X22	4992.00	2057535.88	2057535.88	-0.00		
Ultimate Load Combination: U1							
Factored Applied Forces:							
Node	Column Label	Uu	Mux	Gamma_X	Muy	Gamma_Y	
50	C22X22	-684.00	0.0	0.400	-0.0	0.400	
Factored Stress and Capacity:							
Node	Column Label	uu	f'c	Phi*vc	Critical Point X_Offset	Y_Offset	Status
50	C22X22	-137.02	4000.00	189.74	2.00	2.00	Safe

Figure 19 – Two-Way Shear Results around Column (spMats)

Two-way shear check around interior piles:

According to ACI 312-14 (R13.2.7.2), if shear perimeters overlap, the modified critical perimeter should be taken as that portion of the smallest envelope of individual shear perimeters that will actually resist the critical shear for group under consideration. The hand and reference solutions used this provision. [spMats](#) reports standard shear perimeter for three conditions (interior, edge, and corner) only considering adequate spacing and edge distance is provided to prevent overlapping or truncated shear perimeter.

Two-way shear check around corner piles:

For corner piles, the reference and hand calculations used the smaller of the two perimeters in determining the two-way shear. The reference mentioned that this approach is similar to the investigation of two-way shear in two-way slabs where a cantilevered portion of the slab is adjacent to an edge column. [spMats](#) investigate two-way shear around the corner piles using the critical section that is located a distance d/2 from the face of the pile.

For two-way shear check around column, [spMats](#) uses d as the effective depth used in pile cap. For two-way shear check around the piles, [spMats](#) uses d as the thickness of the pile cap minus the pile embedment.

6.3. Pile Reactions

The model results provide a detailed list of the pile reactions indicating the magnitude and direction of the resulting forces on each pile in the foundation model. Whether force is downward compression or upward net tension on the pile, the load combination producing the maximum reaction is denoted in the output results table. As shown in the figure below the pile reactions are essentially uniformly distributed to all 8 piles indicating a very rigid pile cap with closely spaced piles. The maximum service pile reaction is 84.5 kips compare with service load capacity of 110 kips (50 tons). Note that [spMats](#) also provides the piles factored reactions (114 kips) and can be compared with piles factored load capacity if this value is provided.

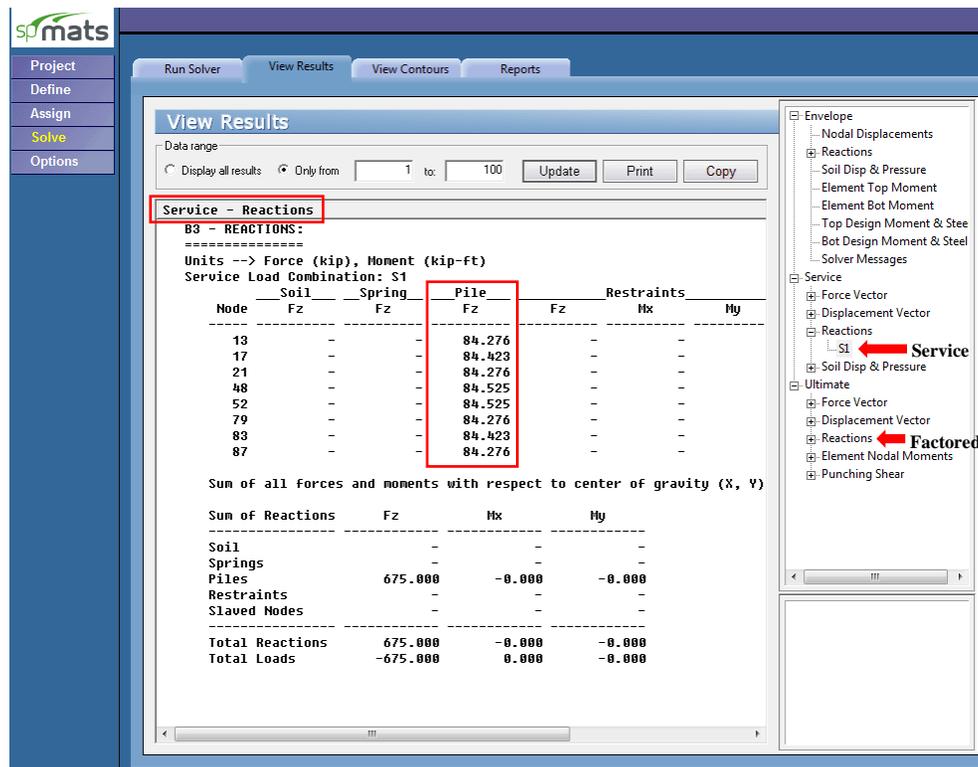


Figure 20 – Piles Service Reactions (spMats)

6.4. Pile Displacement

The model results provide a detailed list of service and ultimate displacement at each node within the foundation model. The nodal displacements indicate the magnitude and direction of the upward to downward nodal movement resulting in the pile cap including nodes where piles are present. It is very essential to determine pile displacement in order to conform to geotechnical specification or communicate with pile contractor. As shown in the figure below the service displacements throughout the pile cap are uniform including at pile locations. This is largely attributed to the high pile cap thickness used in this example making the pile cap translate as a rigid body. Another observation can be made based on the type of soil and corresponding pile stiffness used. When changing bed rock to sand the settlement increases from 1.2 in. to 6.6 in. This indicates, the pile stiffness value highly affects the pile cap settlement. It is, therefore, recommended that pile or soil stiffness values be

obtained from a geotechnical design professional along with maximum permissible displacements at service and ultimate load levels to optimize the foundation model and corresponding design results.

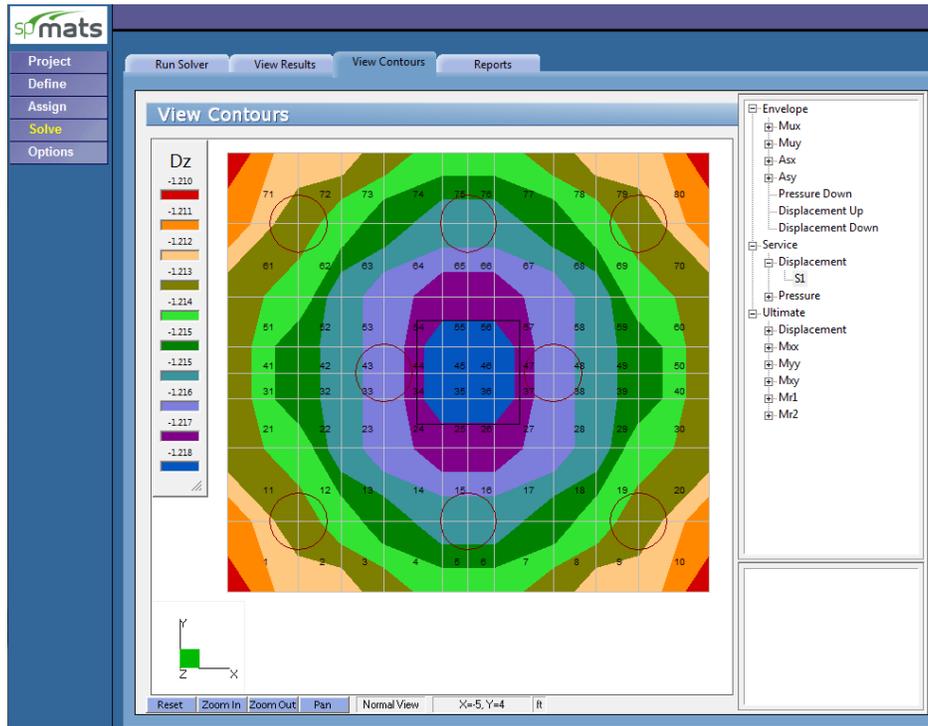


Figure 21 – Vertical Displacement Contour using Bed Rock (spMats)

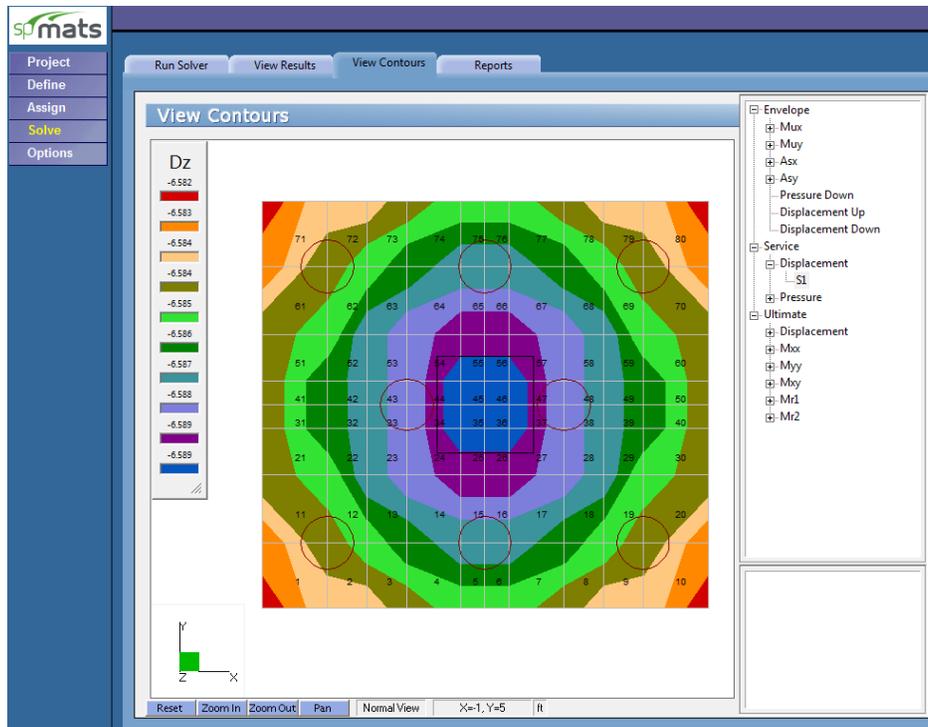


Figure 22 – Vertical Displacement Contour using Sand (spMats)

6.5. Pile Cap Model Statistics

Since **spMats** is utilizing finite element analysis to model and design the foundation. It is useful to track the number of elements and nodes used in the model to optimize the model results (accuracy) and running time (processing stage). **spMats** provides model statistics to keep tracking the mesh sizing as a function of the number of nodes and elements.

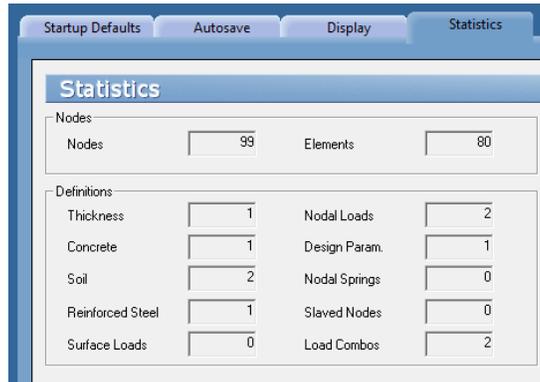


Figure 23 – Model Statistics (**spMats**)

6.6. Column and Pile Design - spColumn

spMats provides the options to export columns and pile information from the foundation model to **spColumn**. Input (CTI) files are generated by **spMats** to include the section, materials, and the loads from the foundation model required by **spColumn** for strength design and investigation of piles and columns. Once the foundation model is completed and successfully executed, the following steps illustrate the design of a sample pile and column.

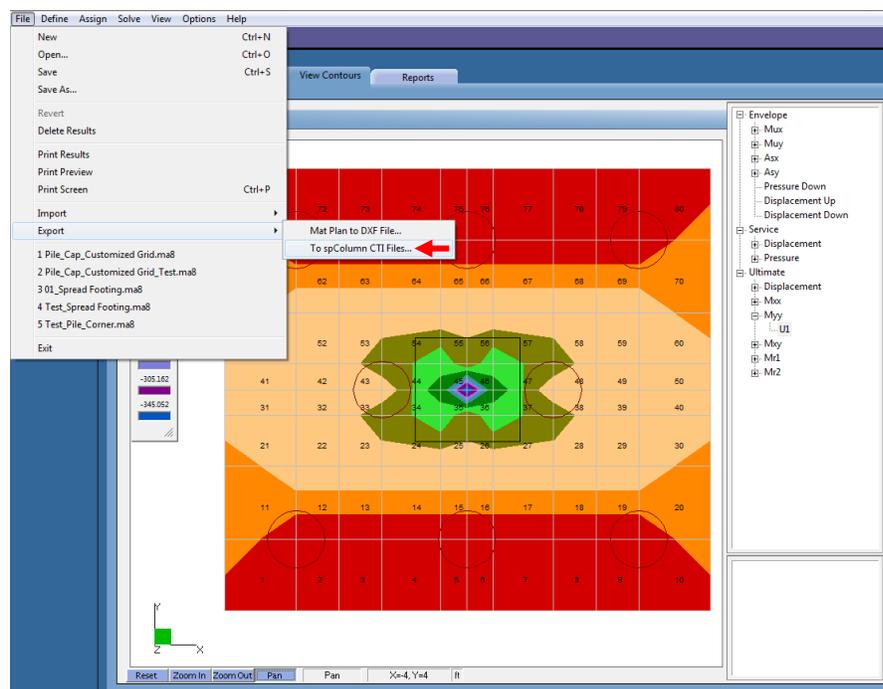


Figure 24 – Exporting CTI Files (**spMats**)

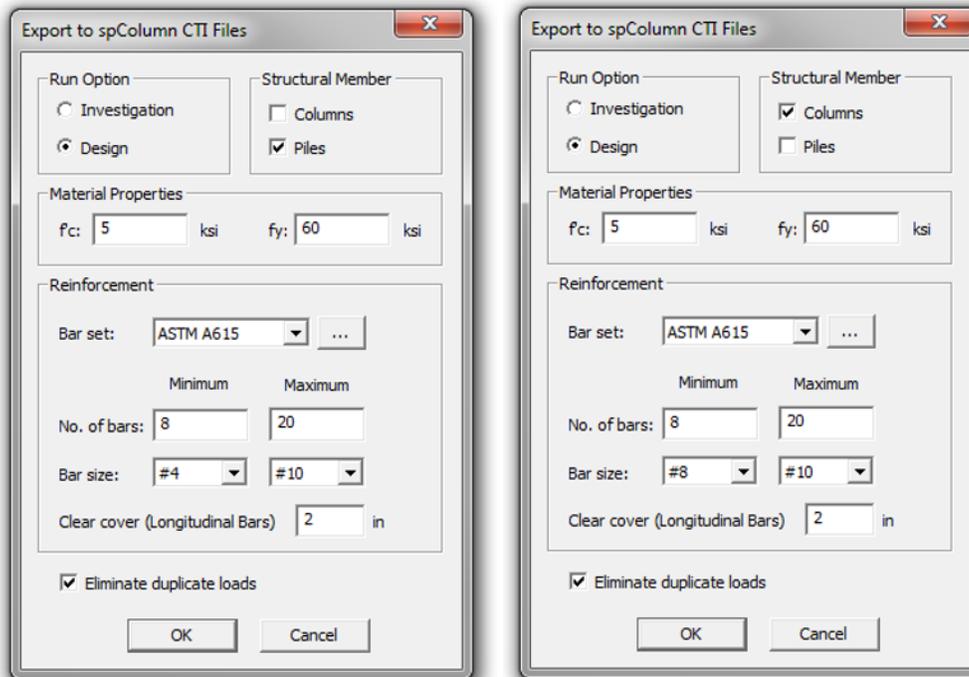


Figure 25 – Exporting CTI Files Dialog Box (spMats)

After exporting spColumn input files, the pile and column design/investigation can proceed/modified to meet project specifications and criteria. In the following a sample pile and column design results are shown as an example.

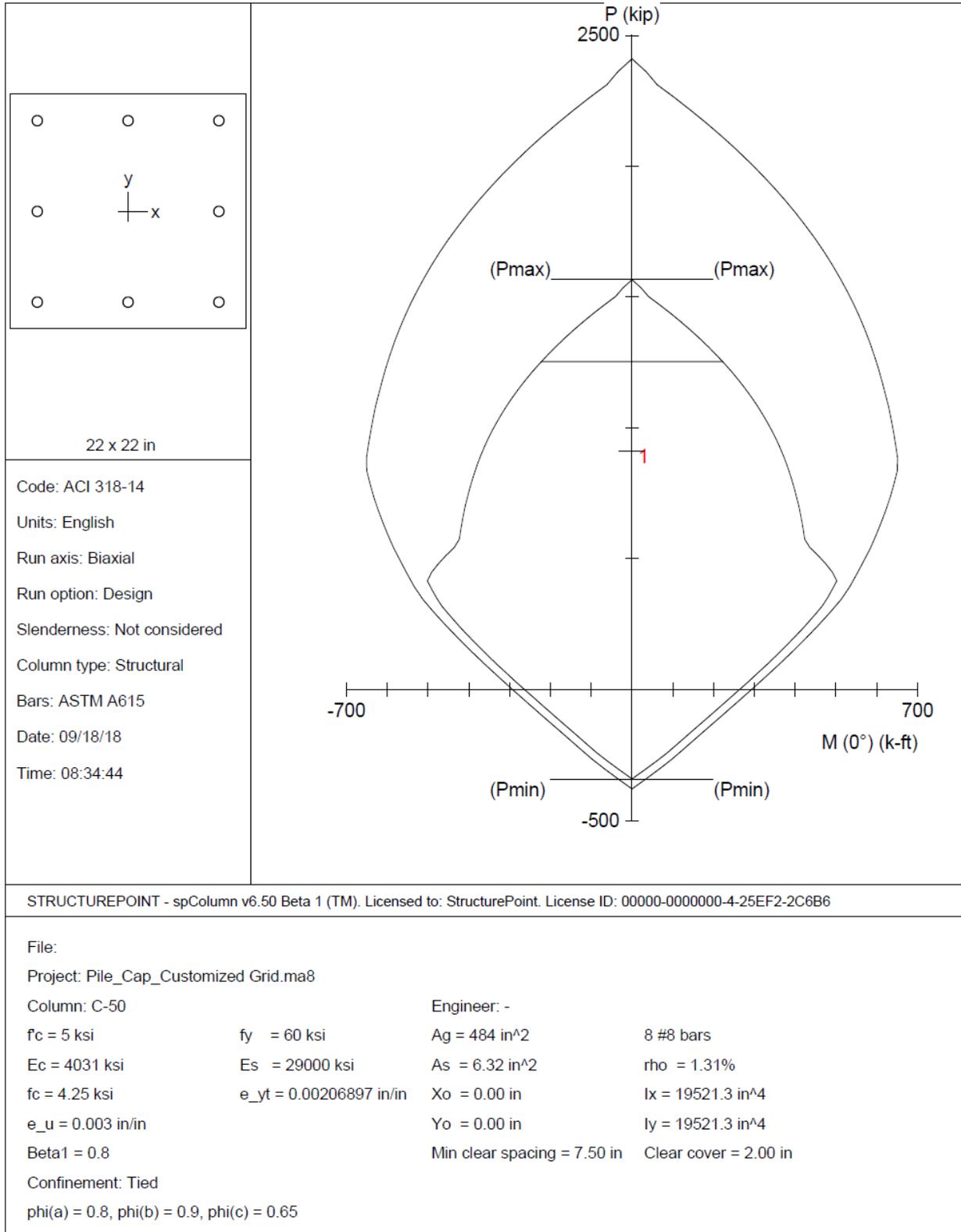


Figure 26 – Column Interaction Diagram with Factored Load (spColumn)

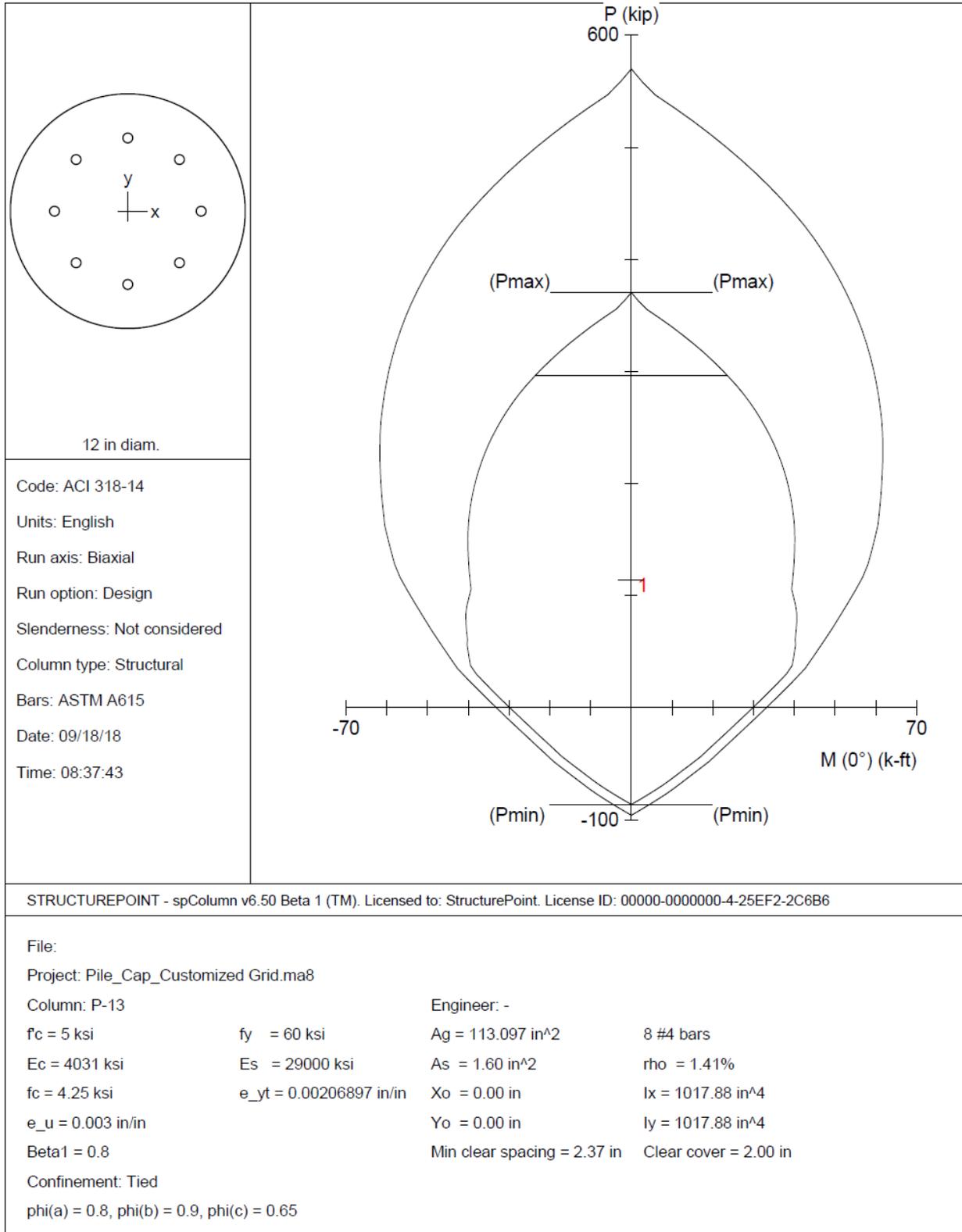


Figure 27 – Pile Interaction Diagram with Reaction Applied (spColumn)

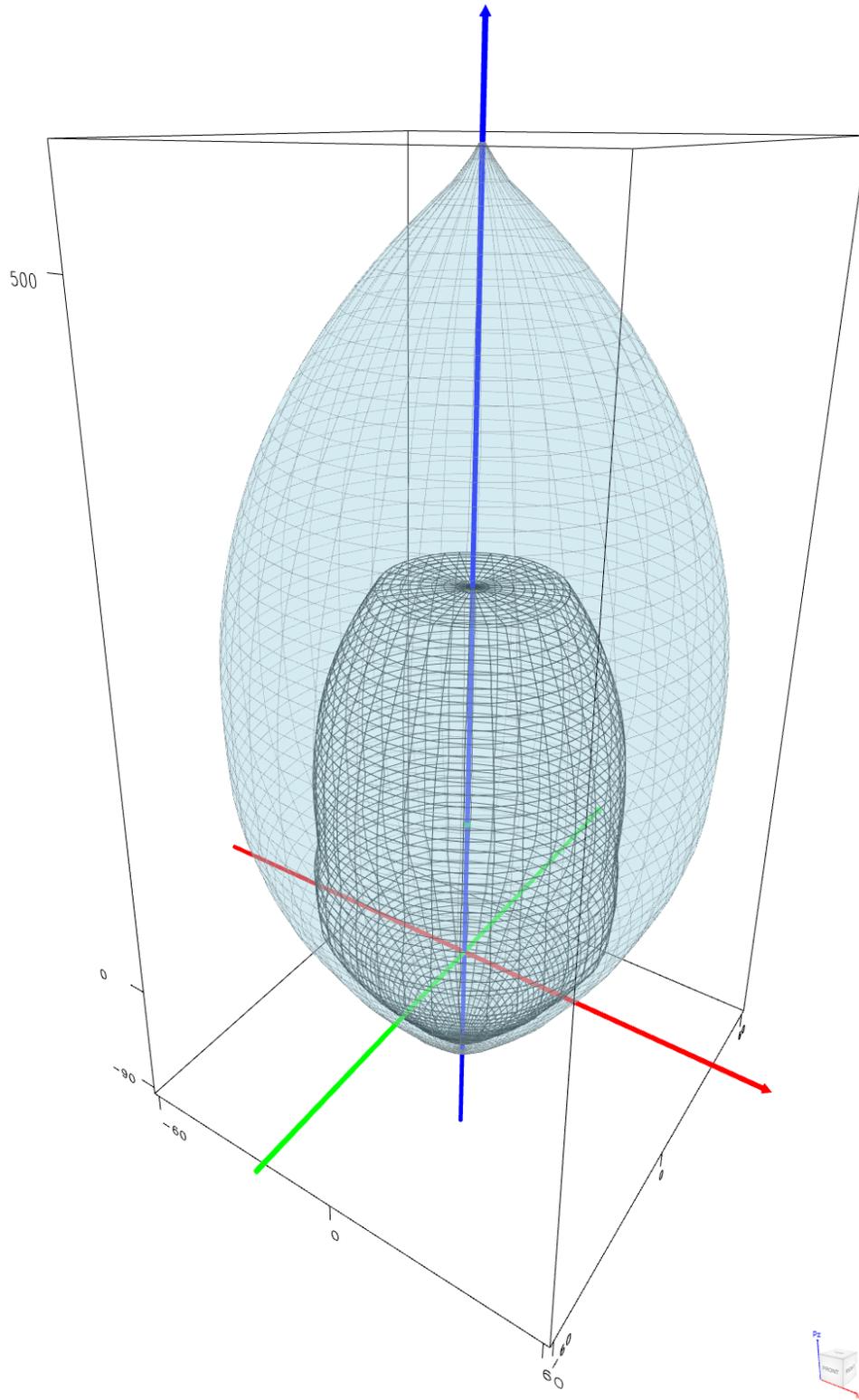


Figure 28 – Pile 3D Failure Surfaces (spColumn)

2D/3D Viewer is an advanced module of the [spColumn](#) program. It enables the user to view and analyze 2D interaction diagrams and contours along with 3D failure surfaces in a multi viewport environment.

2D/3D Viewer is accessed from within [spColumn](#). Once a successful run has been performed, you can open 2D/3D Viewer by selecting the **2D/3D Viewer** command from the **View** menu. Alternatively, 2D/3D Viewer can also be accessed by clicking the 2D/3D Viewer button in the program toolbar.

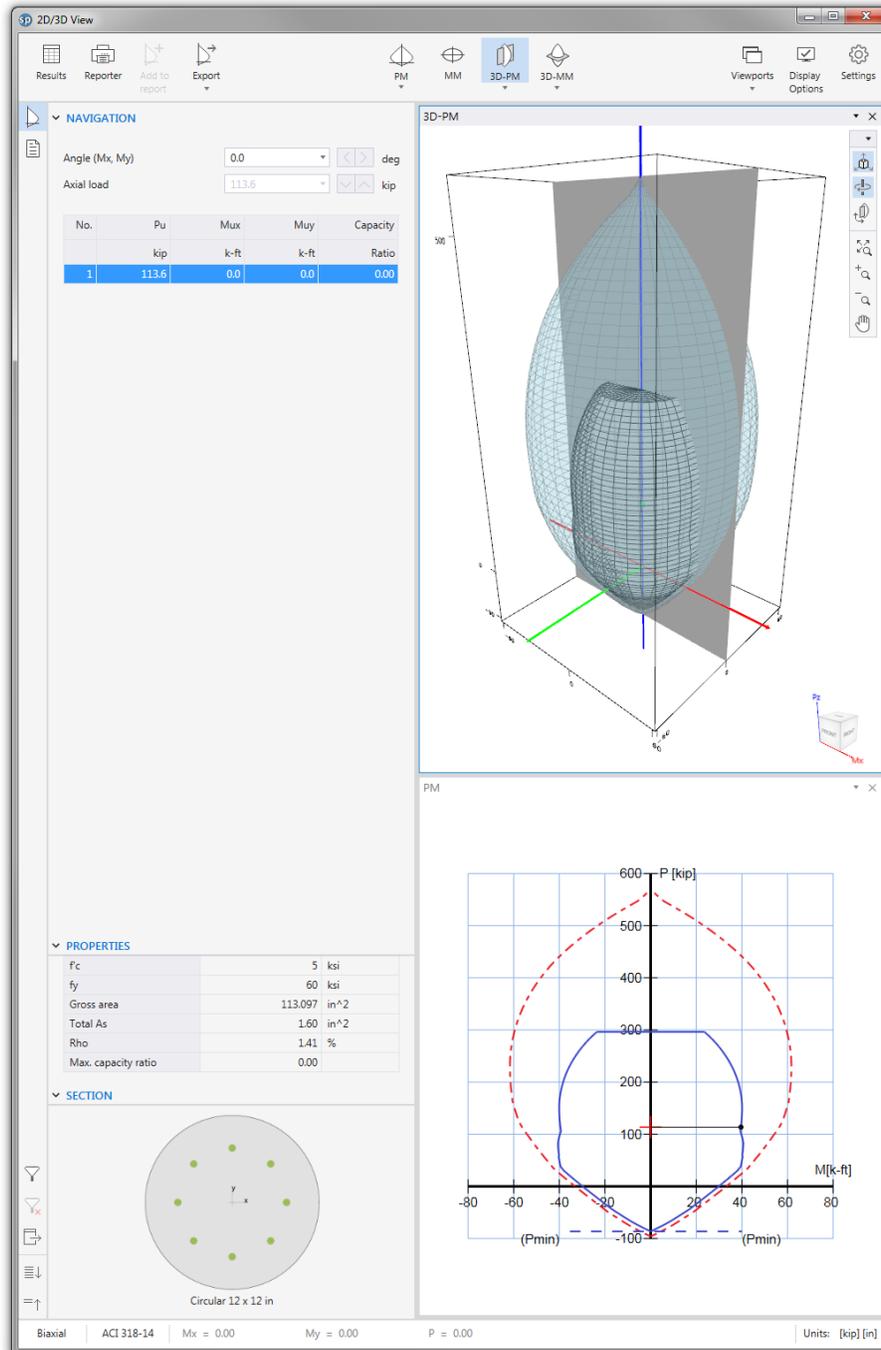


Figure 29 – 2D/3D View for Pile ([spColumn](#))

To further optimize pile design, it was agreed with the builder that 6#4 reinforcement cage can be used for this pile. The following figure illustrate the reduced axial strength capacity is adequate to resist the maximum pile loading.

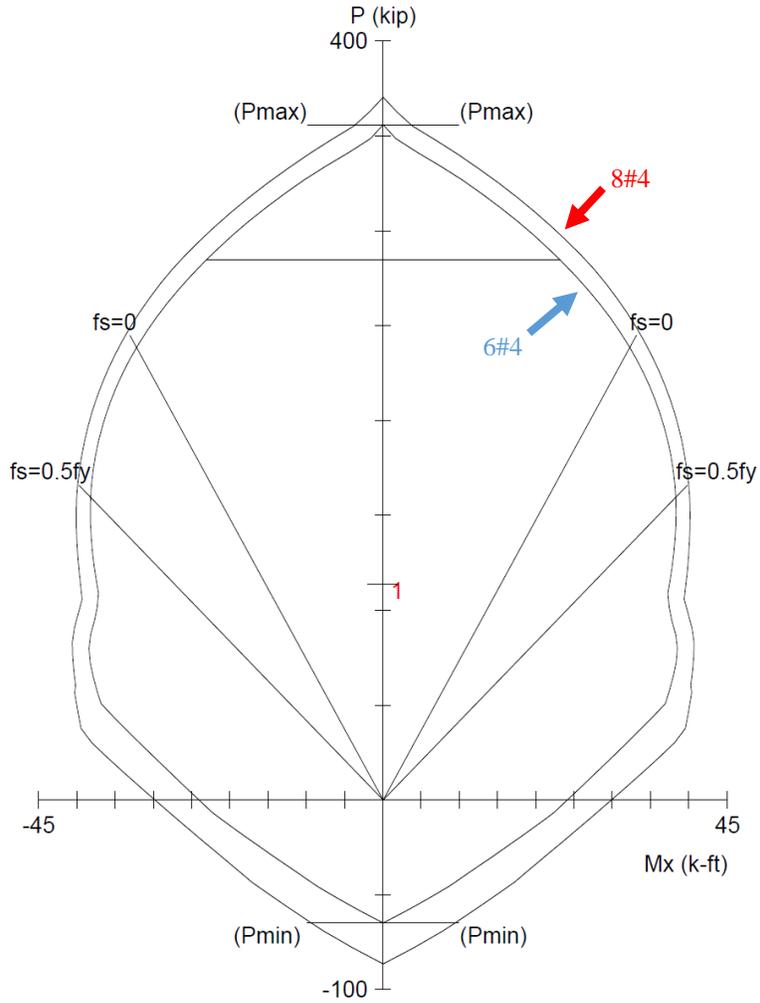


Figure 30 – Superimpose Feature (spColumn)

The builder was provided two options for confinement to increase field and construction flexibility. The impact of spiral vs tied confinement is illustrated below.

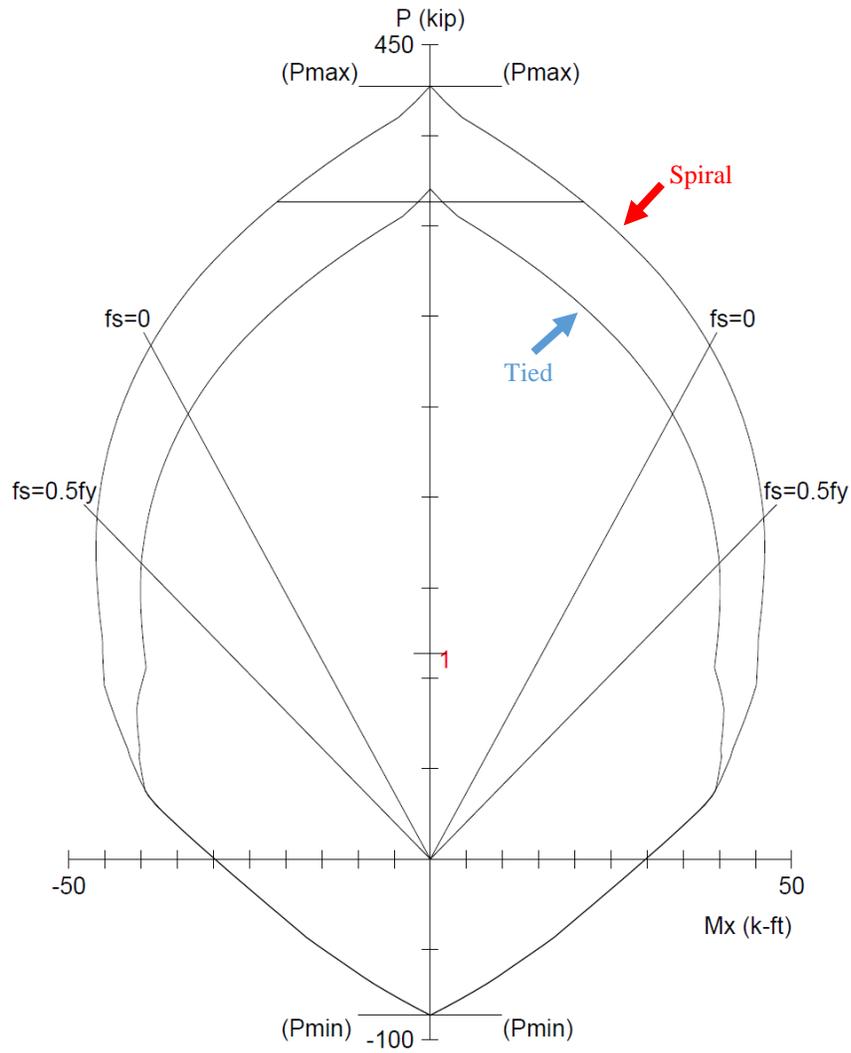


Figure 31 – Tie Confinement vs. Spiral Confinement ([spColumn](#))